Embracing the Dragon’s Tail: Range Estimation of Project Cost and Schedule

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Their judgment was based more on wishful thinking than on sound calculation of probabilities; for the usual thing among men is, when they want something, they will without any reflection, leave that to hope; while they will employ the full force of reasoning in rejecting what they find unpalatable.

- Thucydides

Introduction: The Need for Good Estimates
White-collar project management is beset with problems imposed from the outside: not enough people assigned to the project, over ambitious deadlines, last minute scope changes, and unpleasant technical surprises. But not all problems can be blamed on outside forces; many come from within the project. Those from within frequently originate in a plan that was poorly put together. In such plans lurk: poor recognition of what needs to be included in the technical work, poor assessment of risk, poor estimation of the effort required to get the recognized work done, and poor consideration of how to schedule the effort. Good estimation is one of the key elements for building a robust plan that enables adequate management of the project.

Estimating often begins with a top-level project estimate for the annual budget a year or more before the particular project will start. The estimator usually uses analogy with historical events and the general parameters of the project to arrive at a ballpark estimate. The accuracy of these initial budget estimates can vary as much as ± 40%, but because they stand among other, similarly broad estimates, the assumption is that the later, detailed plans will offset one another and that the total estimate for the year will remain accurate. [1]

More detailed estimates emerge later on, when the full work of the project is being planned. The later estimates do vary in their actual size. Unfortunately, they also vary in their accuracy.

Current Problems
Detailed estimates vary in accuracy because it’s easy to make mistakes. We can forget to include a piece of work in our estimate. We can neglect to include a contingency plan for the risk events that may occur. Concern for the opinion of others can bias our estimate. We can get anchored to our first guess. We can unwisely restrict each detailed estimate to a single value. We can have an unconscious internal bias, either high or low. [2]

Better estimates require that we overcome most of the above problems. In the first half of this article, we will define estimating, suggest techniques that will improve individual activity
estimates, and show how to use these techniques to create robust estimates for the project’s overall cost and schedule. In the second half of the article, we will examine how to apply the theory “in the noisy world” where the plan must support the flexible, profitable execution of the real project.

The Quiet Room: Creating the Project’s Unbiased Estimates

Quiet minds cannot be perplexed or frightened but go on in fortune or misfortune at their own private pace, like a clock during a thunderstorm.  

-Robert Louis Stevenson

Discussions about estimating are frequently interrupted with questions about how the theory will work in practice. Such concerns are serious, but they can be distracting as we concentrate on a new idea. We will put aside these questions for the time being and assume that we will be able to deal with the real world when the time comes. Our first job is to understand the fundamentals as well as we can.

So we make the “quiet room” assumption—until further notice, we close the door on the noisy real world of argumentative bosses, unreasonable price structures, dictated deadlines, and competitive pressures. We don’t want any of reality’s noise to bias our work. In the quiet room, we are all rational, thoughtful individuals, dedicated to generating the best possible estimates of cost and schedule. We are not trying to beat the other golfers, we are trying to beat “old man Par”—the course itself.

A Definition

Before we can improve our project estimates, we need to understand that “an estimate” is a range of values. “The trip will take you two to three hours;” “The sweater costs thirty to forty-five dollars;” “Temperature will be in the 60’s;” all are familiar range estimates from everyday life. When a single value is given as an estimate, it usually has an implied level of certainty about it. For example, “There’s a small chance we could see 70 degrees.” We estimate because we are not certain. We estimate with a range.

Because an estimate commonly means a range of values and not a single point, a single value implies a selection from a range of values and a level of certainty that also comes from that range. If we can develop a better way to handle ranges of values, we can better align our uncertain knowledge of the future with our present need to predict it.

Deming’s Understanding of Variation in Systems

A range estimate allows us to take into account a very important property of a project—the variation in the work. Edwards Deming called this variation “common cause” variation, the day-to-day fluctuations that occur in every process. Deming argued that any real understanding of a system (such as a project) must be based on a fundamental understanding of how it varies. [3] This concern has led to an emphasis on “six sigma” total quality efforts in modern business. For those charged with managing projects, this concern means that project estimation must be built on
an understanding of the variation in the day-to-day work. But before we can discuss variation, we must first identify the work.

**A Foundation of Identified Work**

The fundamental project planning tools identify the work and establish the necessary foundation for project estimating [4].

- The product description defines the attributes of the product of the project.
- The scope statement defines the boundary of the project.
- The work breakdown structure details all the work (the activities) necessary to produce the deliverables of the project.
- The network logic diagram shows the necessary order of the activities and perhaps reveals a few forgotten activities.
- The risk plan adds mitigation activities and an appropriate amount of contingency cost and schedule to the overall project plan. (The risk plan takes account of the other kind of variation in systems that Deming was concerned about—“special cause” variation.)

The fruit of these five analyses is a high probability that we have identified most, if not all, of the pieces of work. With this solid foundation of identified work, we can begin to answer the quantitative questions of “how much?” and “when?” We are ready to estimate.

**Home on the Range**

Looking at our uncertainty, the first range that emerges is the two-point, beginning-to-end, low-to-high range. Once we have declared a two-point range, we set up an variety of possible images.

**Figure 1. Different Interpretations of “From A to B”**

![Figure 1](image-url)

The first example (1) in Figure 1 allows for possible values outside the range (below A and above B). A good practice for developing a clear estimate is to agree that A and B will be the end-points, the extreme values, so that all other possible values lie between them. Under that agreement, the second example (2) in Figure 1 is the best, while the third example (3) should be tightened up to look like the second.

After defining the end points, we should think about how the real world might populate that range if the event happened over and over. We can create a picture, a histogram, a distribution, such as those illustrated in Figure 2.
If we assume that our histograms reflect reality, we have established a picture of the relative likelihood of different values.

The Dragon’s Tail

Experienced individuals favor different work distributions. One veteran project manager assumes that only the beginning and the end of the range are known and that everything in between is equally likely (Histogram 1 in Figure 2). Another imagines the values from A to B are distributed in a normal curve (Histogram 3 in Figure 2). However, many experienced managers of white-collar-work projects have discovered that most work distributions have an interesting property—“the dragon’s tail.”

A long tail of bad experiences that trails out to the high side of the distribution. (Histograms 4 and 5 in Figure 2 [6].)

(The reverse asymmetry can occur when individuals systematically pad their estimates. The points would cluster near B, with a few optimistic events trailing off the low end. But here, in the quiet room, we assume that all estimates are made in good faith, with no padding, and that every effort is being made to eliminate biases.)

While there are a variety of ways to simplify these histograms, one method that combines simplicity with flexibility is to assume that the histogram fits a regular shape, a known distribution, with specific formulas for computing a mean and a standard deviation.

The two most common shapes that are in current use are the “triangular” and the “beta.” Both trace their roots back to work done in the fifties when the critical path method was being developed and when the Navy’s Program Evaluation Review Technique (PERT) was defined [8].
In both cases, after the shape has been agreed to, it can be characterized with the one additional number, the most likely single value, “M,” the highest point in the distribution. Our shape is determined with a three-point estimate.

The three-point estimate makes a number of simplifying assumptions:
1. A is really the lowest possible value.
2. B is really the highest possible value.
3. The points in between have a known shape (probability density function).
4. There is only one high point, M.
5. The range from A to B is continuous.

Details on the Triangular and the Beta Distribution
The similarities and differences between the two distributions are straightforward to list. Both distributions are finite (they have a low end and a high end), both can be used to represent skewed distributions as well as symmetric distributions, and both use the third number to denote the mode (the highest point) of the distribution.

Table 1. Triangular and Beta Distributions Compared

<table>
<thead>
<tr>
<th></th>
<th>Low</th>
<th>High</th>
<th>Likely</th>
<th>Mean</th>
<th>Std.Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangular</td>
<td>3</td>
<td>9</td>
<td>5</td>
<td>5.67</td>
<td>1.25</td>
</tr>
<tr>
<td>Beta</td>
<td>3</td>
<td>9</td>
<td>5</td>
<td>5.33</td>
<td>1.00</td>
</tr>
</tbody>
</table>

From the examples in Figure 3 and Table 1, we can see that the beta distribution bunches more closely around the most likely single value, the mode, while the triangular distribution spreads out more along the full range. Table 1 shows the values for the mean and the standard deviation of both distributions.

Notice that the triangular distribution gives a higher (more pessimistic, more conservative) value for the mean in the example. Also notice that the standard deviation, a measure of the spread, is, as we would expect from looking at Figure 3, broader for the triangular distribution (more uncertainty, more conservative). What may be less clear from Table 1 is that the formulas for the triangular distribution mean and standard deviation are exact, while the formulas commonly used (and used here) for the beta values are approximations of the actual, more complex formulas [9].
**Single-point versus Three-point Estimates**

Before we go on to apply three-point estimates to our real work, we should note that sometimes a single value is a good first approximation of certain ranges of values. When we set out to estimate a piece of work, several considerations come into play before we begin to use our new tool.

- If the work is brief, say a couple of hours, a single point is sufficient.
- If the work is well understood and has little expected variation, say a few hours variance in four days of work, then a single point might be all that’s required.
- If the activity is a few tiny pieces spread out over the calendar, say twelve phone calls to get approval spread out over three weeks, a simple multiple of twelve times the length of the average phone call may be good enough.
- If it’s a meeting that always lasts one hour, a single value may be okay.
- If the work is a known percentage of a calendar work period, say 10% of your daily or weekly work, it may be better to base your estimate on the calendar.
- If the duration of the activity is fixed, for example, paint takes twelve hours to dry, then a single number may be acceptable.
- If experience dictates that an activity always takes six days of calendar time, then a single value for its duration is appropriate.

In sum, where certainty is high and variation is extremely low, a single-point estimate is quick, convenient, and appropriate.

Notice that we can use a single-point estimate with confidence because we implicitly assume it’s really the center (the mean) of a very small range of symmetric values. We have not abandoned our definition of an estimate as a range of values.

Beyond the certainty of a single point lies the realm of uncertain ranges, skewed distributions, and three-point estimates. Let’s review when they become necessary.

- If the work is large, a three-point estimate is probably called for.
- If the work is poorly understood and has considerable variation, for example, plus or minus a day on four days of work, then a three-point estimate is required.
- If it’s a meeting that always lasts from one to three hours, a three-point estimate may be helpful.
- If the work is a variable percentage of a calendar work period, say 5% to 20% of your daily or weekly work, it may be better to use a three-point estimate based on the calendar.
- If experience dictates that an activity takes an indeterminate number of days, say from four to seven days of calendar time, then a three-point estimate of the duration is appropriate.

In short, where certainty is low and variation is high or asymmetric, a three-point estimate is quick, convenient, and appropriate.
The Three-Point Process for a Single Activity
Consider a clearly defined, one- to two-week activity. How long will it really take? How much work is involved? To simplify, let’s assume that one person, working full time, will do the work.

We ask five questions:

1. What’s the most likely single amount of time the job can take? (M)
2. What’s the best time we can reasonably expect to get this job done? (A)
3. What’s the worst time it could reasonably take to get this job done? (B)
4. After considering A and B, what do you now think is the most likely single amount of time the job can take? (M reconsidered)
5. What assumptions did you make as you answered the first four questions?

For example, let’s say you believe an estimated activity will take you about five days. Begin with five and then consider the extremes. How little time could this take if everything goes unbelievably well? Four, maybe as few as three, days. How likely is three? Very unlikely, but still reasonable. But it couldn’t get be done any faster than that. So the low value is a three.

What’s the worst case? Nine, perhaps ten days. That’s with everything going wrong. What about a “re-do” because the approach to the problem doesn’t work? Better push that up to twelve days. Does twelve sound like a reasonable upper bound? Yes. Twelve it is.

Revisit your original five-day estimate for the most likely. Is it really the most likely single value? Now it looks a little too close to the three. Maybe six days would be a little more reasonable. Does that feel about right? Six, yes, let’s say six.

You have arrived at a 3-6-12 estimate. Note that the proportional result is an accident. You conducted three independent evaluations to arrive at three good estimates: 3, 6, and 12. With the effort estimate complete, you have declared what the activity is worth, what you expect to put into the effort, and what the activity will cost in staff-time or dollars.

Next you must estimate how much calendar time, or duration, the activity will require. If the task is performed by one-person, full-time, you’re done. Effort and duration are the same. But what if the task must be done by person who can only work part time? You might estimate the duration as follows.

Three-point Duration
Begin with the most likely single duration. You know it’s six days of work, but you also know you can’t get more than half your time free to work on this activity. That means the most likely single value looks like twelve or more calendar days.

The low of three translates into six calendar days at a minimum. But you remember that there is a chance you could devote more than a half-time effort when you begin the work. So you reduce the low estimate to five calendar days.

You consider the worst case. Twelve full days expands to at least twenty-four calendar days. You know you may not get to devote as much as half time for that long a period, so it could get even
longer. You also think about interruptions to the work and you decide the worst case is closer to thirty calendar days.

Looking again at the most likely single value, you decide that thirteen is a good value for the most likely single number of calendar days. You have arrived at a 5-13-30 calendar-day estimate of the duration of the activity. You write down your list of assumptions so you won’t forget them [10].

**Combine Estimates by Using the Mean and the Variance**

If we are going to combine a single activity’s three-point estimate with others, we need to know how. The method is simple—add up the means and add up the variances. (Note that we can include *any* range estimate—not just a 3-point one—as long as we know its mean and variance.)

To begin, we need to know how to figure the mean and the variance for our three-point estimate.

For a triangular distribution (the more conservative distribution and the one with the exact formulas) the two equations are [11]:

Mean = $\frac{(A + M + B)}{3}$
Variance = $\frac{(B - A)^2 + (M - A)(M - B)}{18}$

The standard deviation of any activity, or of any total, is the square root of the variance.

For our triangular distribution, the mean is the weighted center of the triangle. For our 3-6-12 distribution, the weighted center is 7 staff-days. More than half the values are less than 7, but when the value is above 7 it is more likely to be far above.

The standard deviation is the average spread around the weighted center, in this example, $\sqrt{\frac{(81-18)/18}{18}} = \sqrt{3.5} = 1.9$ staff-days. (For the 5-13-30 duration, the mean is 16 days, the standard deviation is 5.2 days.)

**Combining Estimates**

Table 2 below shows how range estimates add up. We can see how the individual means and variances add up to the total mean and variance. We can also see how the standard deviation, the measure of the spread of the estimate, is calculated as the square root of the variance, both for the individuals and for the total [12].

As we study the table, notice a most interesting property of the built-up estimate—the whole is more accurate than the parts of its sum! While the spread (the standard deviation) around the mean of each individual piece of work is often over 20% of the mean, the spread of the total of seven activities is under 10%! As we add up pieces, the individual variations offset one another and the resulting total variation is a smaller percentage of the total mean. (The square root function makes the total’s standard deviation grow more slowly than the total mean [13].)
Table 2. Range Estimates Added Together

<table>
<thead>
<tr>
<th>ID</th>
<th>Name</th>
<th>Low</th>
<th>Most Likely</th>
<th>High</th>
<th>Mean</th>
<th>Std.Dev</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.2</td>
<td>Initial draft</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.2.1</td>
<td>Gather information</td>
<td>15.0</td>
<td>25.0</td>
<td>45.0</td>
<td>28.3</td>
<td>6.2</td>
<td>38.9</td>
</tr>
<tr>
<td>2.2.2</td>
<td>Write sections</td>
<td>15.0</td>
<td>30.0</td>
<td>60.0</td>
<td>35.0</td>
<td>9.4</td>
<td>87.5</td>
</tr>
<tr>
<td>2.2.3</td>
<td>Review informally</td>
<td>5.0</td>
<td>10.0</td>
<td>25.0</td>
<td>13.3</td>
<td>4.2</td>
<td>18.1</td>
</tr>
<tr>
<td>2.3</td>
<td>Inspection cycles</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.3.1</td>
<td>Inspectors inspect</td>
<td>5.0</td>
<td>10.0</td>
<td>25.0</td>
<td>13.3</td>
<td>4.2</td>
<td>18.1</td>
</tr>
<tr>
<td>2.3.2</td>
<td>Prepare defects/issues list</td>
<td>10.0</td>
<td>20.0</td>
<td>40.0</td>
<td>23.3</td>
<td>6.2</td>
<td>38.9</td>
</tr>
<tr>
<td>2.3.3</td>
<td>Resolve defects/issues</td>
<td>20.0</td>
<td>30.0</td>
<td>40.0</td>
<td>30.0</td>
<td>4.1</td>
<td>16.7</td>
</tr>
<tr>
<td>2.3.4</td>
<td>Make necessary changes</td>
<td>5.0</td>
<td>10.0</td>
<td>25.0</td>
<td>13.3</td>
<td>4.2</td>
<td>18.1</td>
</tr>
</tbody>
</table>

Totals | 156.7 | 15.4 | 236.1

The fact that the whole is more accurate than the parts of its sum is the fundamental reason that bottom-up, range-based estimates give such useful results.

- The total includes our best thinking on each small range where we are most likely to be accurate.
- The total includes our understanding of the “common cause” variation in the individual pieces of work.
- The arithmetic of the addition of variation gives us an increasingly accurate sum, a sum with a decreasing percentage standard deviation.
- The decreasing percentage standard deviation has the effect of squeezing any “padding” in the parts by recognizing that the parts’ variations largely offset each other.

These effects are all apparent in the evolving, or “rolling wave” estimate associated with a large project that is shown below.

Distributions from Triangular to Normal
Another interesting feature of the combined range estimates is that, no matter what the shape of the distributions of the pieces (like our triangles), by the time we have added three or four individual pieces, the distribution of the sum approximates a “normal” distribution (see Figure 4)[15].

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The area under the normal, or “bell,” curve is broken up into known percentages between the standard deviations (see Figure 4 and Table 6). The mean of the sum is in the middle. Half the time the total will be more; half the time, less. To be 50% sure, we can select the mean. To be 84% sure, we select the first standard deviation above the mean; for 98%, the second standard deviation.

**Table 6. Normal, “Bell” Curve Standard Deviations and Cumulative Likelihood (Chance)**

<table>
<thead>
<tr>
<th>Std. dev.s above mean</th>
<th>Chance</th>
<th>Std. dev.s above mean</th>
<th>Chance</th>
<th>Std. dev.s above mean</th>
<th>Chance</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>50%</td>
<td>1.0</td>
<td>84%</td>
<td>2.0</td>
<td>97.7%</td>
</tr>
<tr>
<td>0.1</td>
<td>54%</td>
<td>1.1</td>
<td>86%</td>
<td>2.1</td>
<td>98.2%</td>
</tr>
<tr>
<td>0.2</td>
<td>58%</td>
<td>1.2</td>
<td>88%</td>
<td>2.2</td>
<td>98.6%</td>
</tr>
<tr>
<td>0.3</td>
<td>62%</td>
<td>1.3</td>
<td>90%</td>
<td>2.3</td>
<td>98.9%</td>
</tr>
<tr>
<td>0.4</td>
<td>66%</td>
<td>1.4</td>
<td>92%</td>
<td>2.4</td>
<td>99.2%</td>
</tr>
<tr>
<td>0.5</td>
<td>69%</td>
<td>1.5</td>
<td>93%</td>
<td>2.5</td>
<td>99.4%</td>
</tr>
<tr>
<td>0.6</td>
<td>73%</td>
<td>1.6</td>
<td>95%</td>
<td>2.6</td>
<td>99.5%</td>
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<tr>
<td>0.7</td>
<td>76%</td>
<td>1.7</td>
<td>96%</td>
<td>2.7</td>
<td>99.7%</td>
</tr>
<tr>
<td>0.8</td>
<td>79%</td>
<td>1.8</td>
<td>96%</td>
<td>2.8</td>
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</tr>
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<td>0.9</td>
<td>82%</td>
<td>1.9</td>
<td>97%</td>
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</tr>
<tr>
<td>1.0</td>
<td>84%</td>
<td>2.0</td>
<td>98%</td>
<td>3.0</td>
<td>99.9%</td>
</tr>
</tbody>
</table>

**Confidence, Precision, and “Five Sigma” Values**

We see that “confidence” and “precision” are different facets of the same standard deviation. Our confidence in any one overall estimate is determined by how many standard deviations away from the mean it is (and in which direction). Those below the mean have less than a 50% chance of happening (lower confidence); those above, a greater chance (a higher confidence) of coming true.

We have settled on a two-standard-deviation-above-the-mean value for a 98% confidence in the result. Although the curve extends infinitely above and below the mean, for our purposes three standard deviations above and below (“six sigma”) will approximate (99.8%) the entire distribution. When we use two-standard deviations above the mean, we call our plan a “five sigma” plan (three below and two above).
“Five sigma” is intended to indicate that the estimate lacks the precision of the “three-sigma (plus or minus)” control charts or the “six sigma (plus or minus)” quality controls found on some manufacturing floors. Less accuracy seems appropriate because, in the roughly approximate world of projects, everything is much less certain. On the other hand, “five sigma” should indicate that the estimate is made with the same kind of concern for capturing the variation of the real system that has inspired the quality movement in business for the past fifty years.

The “precision” of an estimate is determined by how closely the distribution clusters around the mean, how little the distribution varies, and how narrow is the standard deviation. The standard deviation’s percentage of the mean measures the precision, or accuracy, of the overall estimate (the mean). As we have seen in previous examples, the rule of thumb is: the greater the number of parts added up, the greater the precision of the sum. A project with over forty parts commonly has a standard deviation that is less than 4% of the project mean.

**Single Point Estimates Go Wrong**

Notice that if we had only used a single number for each of our task estimates, we would have been quite wrong in our overall estimate. In Table 5, using the middle number of our three-point estimate as an approximation to the one-point estimate (actually, our middle number may be a bit higher than a quickly considered one-point estimate), we find that they add up to an overall estimate of 162 in Table 5, while the mean of the range estimate is 189.

The single-point overall estimate of 162 is \((189-162)/9 = 3\) standard deviations low! In fact, it is on the border—not even inside—the “six sigma” range of likely outcomes. The likelihood that 162 would come true is one in a thousand. There is a 99.9% chance the estimate is too low!

We can safely conclude that:

The failure to estimate the “common cause” (variation in a project) commonly causes the project estimate to fail.

**“Rolling Wave” Successive Refinement**

On large projects, if we try to estimate the poorly understood work of next year in as much detail as the clearly understood work of tomorrow, we will waste time making estimates that will need to be revised later. A solution to this problem is to estimate the short term on a small scale, the mid term on a larger scale, and the long term on a grand scale [14].

After we have completed the initial estimate, we must later refine that estimate as the mid-term work comes into sharper focus and becomes the near-term work. This kind of estimating further elaborates the detail of the work without enlarging the total size of the work.

In Tables 3, 4, and 5, we see an illustration of a “rolling wave” where we periodically refine our work estimate (in staff days) as the project goes forward. For large projects rolling wave is a “best practice” because it balances our need to estimate with the limits of our insight into far-off work. Notice that the illustrated project’s total estimate does not significantly change as its estimates are successively refined.
Table 3. Rolling Wave Estimate at the Beginning

<table>
<thead>
<tr>
<th>Rolling Wave Estimate</th>
<th>Calculations</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Low</td>
</tr>
<tr>
<td>Near term</td>
<td></td>
</tr>
<tr>
<td>Activity 1</td>
<td>3</td>
</tr>
<tr>
<td>Activity 2</td>
<td>2</td>
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<td>4</td>
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<td>1 term total</td>
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<tr>
<td>Mid term</td>
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</tr>
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<td>7</td>
</tr>
<tr>
<td>Activity 10</td>
<td>5</td>
</tr>
<tr>
<td>Activity 11</td>
<td>5</td>
</tr>
<tr>
<td>Term Total</td>
<td></td>
</tr>
<tr>
<td>Far term</td>
<td></td>
</tr>
<tr>
<td>Activity 12</td>
<td>20</td>
</tr>
<tr>
<td>Activity 13</td>
<td>18</td>
</tr>
<tr>
<td>Term Total</td>
<td></td>
</tr>
<tr>
<td>Project total</td>
<td>18</td>
</tr>
</tbody>
</table>
To accomplish this, we must consider the network of project activities. Every network is made up of path segments. We can add up the individual task durations to arrive at a path segment’s duration, then assemble these path segments to correctly predict a schedule.
Adding Up Durations
Using the same arithmetic that we used to add up the staff days, we can add up the calendar days of duration for a string of tasks on a particular path. When we draw a picture of a path with the activities scaled to their mean duration, we obtain a picture like the one shown in a in Figure 5 (a time-scaled network diagram).

Figure 5. Triangular Task Durations Add Up to a Bell-Shaped Path Duration

Figure 5 shows that the ending boundary of the fifth task is both at the mean of the last triangular task distribution and at the mean of the bell curve of the path duration distribution. All our discussions about the accuracy and precision of our earlier estimates also apply to the path duration and to the overall project duration (the schedule).

Figure 6 shows a time-scaled network diagram of the fundamental building block of all complex networks—a simple parallel path structure. For simplicity’s sake, each task depicted is staffed with one full-time person, so duration and effort are the same. The structure diverges into two parallel paths that eventually rejoin and come to an end. This simple network example clearly shows how, because of the parallel activity, the budget for the project in staff days is almost always greater than the schedule in calendar days.

Figure 6. A Simple Time-Scaled Network Structure

In the project network depicted in Figure 6, the short parallel path is (has a mean of) 37 days; the long parallel path, 44.4 days. The critical path is 60 days (0.4 days of slack included) and a standard deviation of 6 days. The buffers (slack) where the parallel paths merge are 7.8 days and 0.4 day [16]. The 98% sure schedule for the whole project network is 60 + 2 x 6 = 72 days.

The Figure 6 project cost was reported as 96.6 staff days with a standard deviation of 7.6 staff days; the 98% sure budget is 96.6 + 2 x 7.6 = 111.8 staff-days.
A Rule of Thumb

If we worked through the example in Figure 6, we would notice that, while the cost estimate is usually one big addition for the whole project, the schedule is a series of small additions of path lengths. A convenient rule of thumb for the standard deviation of a path’s length (schedule) would speed up calculating the network.

It turns out that, no matter what the shape of the distribution, if we assume that an average task is about a week long, with a day of standard deviation, we arrive at the following rule of thumb:

When the standard deviation of the weekly path is one day, the standard deviation of the monthly path (four weeks) will be two days, and the standard deviation of the quarterly path (thirteen weeks) will be a little less than four days.

Table 7. Path Standard Deviations

<table>
<thead>
<tr>
<th>Num. of Pieces of Work</th>
<th>Mean</th>
<th>Variance</th>
<th>Std. Dev.</th>
<th>Std. Dev./Mean (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (week)</td>
<td>5</td>
<td>1</td>
<td>1</td>
<td>20%</td>
</tr>
<tr>
<td>4 (month)</td>
<td>20</td>
<td>4</td>
<td>2</td>
<td>10%</td>
</tr>
<tr>
<td>13 (quarter)</td>
<td>65</td>
<td>13</td>
<td>3.6</td>
<td>6%</td>
</tr>
</tbody>
</table>

So, a busy planner can quickly approximate a path’s standard deviation—if it’s one day for a weekly path, then it’s two days for a monthly path, and four days for a quarterly path.

While these rules of thumb apply to any shape of distribution with a 5-day mean and a 1-day variance, one triangular duration estimate that yields a mean of five and a standard deviation of one is near 3-4-8 (rounded to whole days). Figure 7 uses this special 3-4-8 distribution to illustrate the calculations behind the rules of thumb. (The 13th week shows the actual, unrounded, 3-point estimate used in the spreadsheet. The numbers displayed elsewhere in the spreadsheet are rounded to whole days.) The standard deviation of one day in five becomes two days in twenty and four days in 65. Most paths in a project can be approximated with these lengths [17]. For any special problems, the exact values can be computed.
Deming Revisited

Our estimation efforts have allowed us to incorporate Deming’s “common cause” variation for both cost and schedule. His “special cause” variation was assumed in our initial risk plan. After reviewing the risk plan for a second time, we add its contingencies (means and variances for cost risk and schedule risk) to our overall estimates to arrive at the final answer to the questions “How much?” and “When?” Notice that it is possible for us to declare the full “plus and minus three standard deviation” range for both our cost and our schedule. We understand “six sigma” estimates for our project even if we chose to use “five sigma” estimates in practice.

Having answered our initial questions in a quiet room where everyone behaved in a rational and deliberate manner, we now turn our attention to the noisy” real world.

The Noisy World: Using Range Estimates in Everyday Life

A theory must be tempered with reality.

-Jawaharlal Nehru

A big obstacle to discussions of estimation is the controversy it generates about the way the “real” world works. These discussions are difficult because everyone can be right under certain conditions. Practices that work smoothly in one industry would never be tried in another. White-collar and blue-collar work have many different assumptions, both large and small. A 300-person project to build a jet engine has a different set of concerns from a 4-person project to “re-

![Figure 7. Some Duration Rules of Thumb](attachment:image.png)

<table>
<thead>
<tr>
<th>Name</th>
<th>Low</th>
<th>Most Likely</th>
<th>High</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Variance</th>
<th>Standard Deviation as a percentage of the Mean.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Month One</td>
<td></td>
<td></td>
<td></td>
<td>5</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Activity 1</td>
<td>3</td>
<td>4</td>
<td>8</td>
<td>5</td>
<td>1</td>
<td>1</td>
<td>1/5 = 20% Week</td>
</tr>
<tr>
<td>Activity 2</td>
<td>3</td>
<td>4</td>
<td>8</td>
<td>5</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Activity 3</td>
<td>3</td>
<td>4</td>
<td>8</td>
<td>5</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Activity 4</td>
<td>3</td>
<td>4</td>
<td>8</td>
<td>5</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Monthly Total</td>
<td></td>
<td></td>
<td></td>
<td>20</td>
<td>2</td>
<td>4</td>
<td>2/20 = 10% Month</td>
</tr>
<tr>
<td>Month Two</td>
<td></td>
<td></td>
<td></td>
<td>5</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Activity 5</td>
<td>3</td>
<td>4</td>
<td>8</td>
<td>5</td>
<td>1</td>
<td>1</td>
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<td>4</td>
<td>8</td>
<td>5</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Activity 8</td>
<td>3</td>
<td>4</td>
<td>8</td>
<td>5</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Monthly Total</td>
<td></td>
<td></td>
<td></td>
<td>20</td>
<td>2</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Month Three</td>
<td></td>
<td></td>
<td></td>
<td>5</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Activity 9</td>
<td>3</td>
<td>4</td>
<td>8</td>
<td>5</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Activity 10</td>
<td>3</td>
<td>4</td>
<td>8</td>
<td>5</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Activity 11</td>
<td>3</td>
<td>4</td>
<td>8</td>
<td>5</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Activity 12</td>
<td>3</td>
<td>4</td>
<td>8</td>
<td>5</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Activity 13</td>
<td>3.000</td>
<td>4.265</td>
<td>7.735</td>
<td>5</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Monthly Total</td>
<td></td>
<td></td>
<td></td>
<td>25</td>
<td>2</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Quarterly Total</td>
<td>62</td>
<td>4</td>
<td>13</td>
<td>4/65 = 6%</td>
<td>Quarter</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

© 2008-2013 New Leaf Project Management  Embracing the Dragon’s Tail  (Earn PDUs for reading this article)  16  You may not post the PDF to a web page. You may, however, link to its source:  www.newleafpm.com
engineer” an office procedure. (The remarkable achievement of the PMBOK Guide is that it actually found the commonly agreed-upon core within this diversity of activity.)

Compounding the confusion is the spotty data that underlies many statements about estimation. Research in the social sciences is difficult at best and conclusive data to support many claims remains elusive. The careful manager should keep a firm grip on common sense (and a sense of humor) and take all radical assertions with a large grain of salt.

**Individual Differences in Estimating**

There are as many different noisy distractions in the real world as there are individuals to create them. Generalizations about individual behavior tend to reflect the prejudices of the speaker rather than carefully assembled data. Individuals who are anxious to be successful are suspected of padding everything and *overestimating* their work. Those who want to be regarded (consciously or unconsciously) as heroic high-performers might be thought to severely *underestimate* the work [18].

The truth is that these and many other individual attitudes and practices exist. The practical lesson of this truth is that anyone estimating a project **must be on the lookout for biases and be ready to adjust for the biases that are detected**. Range estimates allow careful individuals enough latitude that in many cases they will, over time, begin to correct for the biases they detect in their own earlier work.

**Controlling Padded Estimates**

The problem of padding estimates is alleviated, if not solved, by the way range estimates add up. The manager can allow the wide padding that leads to broader ranges and even some self-serving pessimism where the most likely value appears near the high end, above the mean. For control, the manager has the narrowing effect of the sum’s variance. Because the variances in the low-level estimates are independent of one another, they will generally offset one another, so the sum has a standard deviation that is a smaller percentage of the mean. The overall estimate squeezes the variation and challenges the team to achieve a focused goal.

The manager can detect (and adjust for) a high pad by noting individual performance that compares too favorably with the plan, which always beats the mean [19]. (See more in the discussion below.)

**Assigning Work**

Different organizational experiences, personal styles, and deeply held opinions lead to different ideas about how to work should be assigned. Each opinion has an ardent proponent and work assignments must often be negotiated.

Reliable data show at least two distinct styles of negotiator—the aggressive (25% of the population) and the cooperative (64% of the population with 11% indeterminate) [20]. If we understand our counterpart’s negotiating style, we can better negotiate a successful assignment of work.

When dealing with a fundamentally cooperative negotiator, **we may divulge the full range of the estimate and set the mean as a stretch goal**. (Remember, we expect to hit this number only fifty
percent of the time in a well-done estimate.) We expect the individual worker to stay in touch and turn in work as soon as it is completed, early as well as late. Through staying in touch, we can successfully manage the start of the successor activity.

When dealing with an aggressive negotiator (who will try to take advantage of any range of values), we must adopt a much more cautious approach to our work agreement. Note that while the estimate may be based on a range, **it is perfectly appropriate to negotiate a single deadline with certain individuals** [21]. Getting early work turned in and staying in touch will also be more challenging with an aggressive negotiator.

In every case our behavioral goal is **to be sure that everyone turns in work as soon as it has been completed**, early, on time, or late.

**Work Environments**

An individual’s ability to complete work is greatly influenced by the work environment. The focus of a full-time project assignment can increase an individual’s productivity. At the other extreme, a person whose attention is divided among multiple tasks can see personal productivity decrease to zero. (There’s a great deal of evidence that the split-task environment occurs much too often [22].)

An organization that respects well-managed projects, provides adequate staffing, and acknowledges the “quiet” successes of achieved milestones will experience much higher productivity than one that sets unrealistic goals (also known as “aggressive” schedules or “tight” budgets) and reassigns staff to fight fires as projects regularly get into trouble [23].

A project that emphasizes the team’s responsibility to protect the path’s schedule buffer, rather than the variations in individual schedule performance, will be more likely to stay on schedule.

A project that combines good planning and earned-value progress measurement will perform better against estimates than a project that attempts to control with a “budgeted versus actual” fiction.

All of these work-environment issues will influence how to “plan the work” and “work the plan.” All will affect how we estimate and how our estimate is borne out in the work itself. All are better served with range estimates.

**Common Complaints**

After we have produced a plan with a 98% sure budget and a 98% sure schedule (a “five sigma” plan) the three most common responses are: “That’s way too expensive,” “We need it sooner,” and “It needs more features if it’s going to sell.” To deal with these objections we must:

- **Master the details of our project** so we know how the planned details of scope, schedule, and cost relate to one another.
- **Deeply understand the business reasons** for doing our project [24].
- **Attend to our negotiation skills** and exercise them as much as we can. [25].

After we have prepared, the following additional considerations may be of some help.
“That’s way too expensive!”
We know the project’s cost will always be too high in somebody’s eyes. However, unlike the price, which contains a profit margin that can be negotiated, the cost is tied to the necessary work of the project [26]. Wishing the cost were different will not make it so. As the saying goes, hope makes a good companion but a bad guide.

When people express concerns about the cost of the project, encourage them to discuss what parts of the work (and parts of the product) they would like to omit. It’s often a good idea to explore the reasons why the cost is seen as harmful. In the larger world of business value, the project cost often plays a very small part.

“We need it sooner!”
Insistence on an unrealistically aggressive schedule requires the same response as an unrealistically tight budget (see above). It is our responsibility to understand the business need behind the expressed request. Ask yourself whether this is a case of “never enough time to do it right, but always enough time to do it twice.” If the pressure is artificial, focus the discussion on the real business needs.

“It needs more features if it’s going to sell.”
When the stakeholders are demanding more product for their money and time, show them how their money and time will be spent in the plan. Educate them on the realities of the project work. Probe for the business value behind the desired additional features. Distinguish between “nice to have” and “critical to success.” Collect enough data to arrive at a rational agreement on these points. Be willing to reorder the ranking of product features and cut certain ones in order to include others in a thoughtful re-plan.

No single response is the universal answer to any of these objections. Because our careful estimates are the fruit of honest thought, we can refuse to allow anyone to substitute wishing for thinking. We can insist that alternatives be considered as thoughtfully as the original plan and that the new estimates be as carefully derived as the old ones.

The Robust Plan
At this moment, we make a final check with the stakeholders on what a 10% variation in product features, schedule, or cost would mean, so we can adjust the balance of scope, schedule and cost as the project unfolds [27].

The estimates become the plan when our stakeholders all agree. We have dealt with the “noise” of the real world by using the ranges embedded in our original estimates. We have negotiated as skillfully as we can with all concerned, and have arrived at the moment when the plan is approved and the project can go forward.

Thanks to our range estimates, we can begin the project with a robust, “five sigma” plan. It’s a plan that will withstand reality’s shocks without major re-planning, a plan that will reassure our stakeholders as they see scheduled results emerge at planned milestones for estimated costs.
Conclusion

Treating estimates as ranges begins with a common-sense definition of the term—an estimate is a range of values. Ranges can be described with two end points, with a histogram of values, or with a three point triangular distribution. Ranges of values incorporate elusive system variability and create a whole that is more accurate than the parts of its sum.

A range estimate successfully aligns our uncertain knowledge of the future with our present need to predict it. Range estimates of individual durations combine with network logic and the critical path to produce a durable schedule for the project. Convenient rules of thumb can be used to simplify the rapid calculation of network paths. Range estimates will endure in the noisy real world and can encourage cooperative work assignments, better project communications, and prompt reporting.

Range estimates for the project’s budget and schedule generate a solid “five sigma” plan that can withstand the vicissitudes of the real world, exceed the expectations of the stakeholders, and enhance the profits of the organization.

Notes
1. Early estimating errors reported in Boehm, 1981, show error rates of +/- 40%. While the examples are software projects, reports from many clients in other industries over the past ten years suggest that it is reasonable to assume a similar variation holds true for other estimates made for the purposes of budgeting.
2. For an insightful summary of the current understanding of many of our built-in biases see Hammond, Keeney, and Raiffa, 1999.
3. The project equivalents to Deming’s “common” and “special” causes of variation in systems were first pointed out to me by my associate Mark Durrenberger in 1997. Many more details on understanding systems are explained in Deming’s writings. See especially Deming, 1986, and Deming, 1994.
4. These tools are clearly described in the PMBOK Guide. See PMI, 1996, 2000.
5. While the shapes of distributions can vary, range estimation only requires that we understand our distribution sufficiently well to estimate its mean, the weighted center, and its variance (a measure of its variation about the mean). For any histogram of some number of points, n, where each point is x1, x2, x3…xn, the mean, m, is the average of the points and the variance is the sum of the distances to the mean squared (m-xi)2 divided by n-1. Consult any statistics text for further details on these simple calculations.
6. For early examples of this observation, see Levy, Thompson and Wiest, 1963, and Miller, 1962. Some long-tailed histograms can be described using a log-normal distribution.
7. Our work environment (our “noisy” world) will influence the shape of the distribution that we use. See Goldratt, 1997, and Newbold, 1998, for a discussion of padding assumptions.
8. While there are many other useful distributions, the triangular and the beta distribution go back to the early analysis of PERT techniques. See MacCrimmon and Ryavec, 1962, for more on the details behind both distributions.
9. The simplified formula for the “beta” mean, M = (A + 4*M + B)/6, is an approximate formula that is the result of blending a range of solutions to the real cubic equations! This point is further explored in MacCrimmon and Ryavec, December, 1962.
10. An easy-to-read introduction to the mechanics of estimating can be found in Durrenberger, May, 1999. See also, Winning Project Management, 1998.
11. The simplified formulas for the “beta” distribution are: Mean = (A + 4*M + B)/6 and Variance = ((B – A)/6)^2. Again, both formulas are approximations to the real ones for this distribution. Both sets of formulas are listed on page 116 of the PMBOK Guide (PMI, 1996).


13. Roughly stated, the underlying assumptions for adding up the individual distributions is that they are independent, finite distributions, any one of whose variance’s effect on the sum is “asymptotically negligible.” Variations of these assumptions can also be handled due to the achievements of 20th century mathematics. For additional mathematical insights, see details in Zabell, June-July, 1995.


15. This remarkably convenient fact is the product of mathematical work on the Central Limit Theorem dating from the late 1600s through the first third of the 20th century, when a young Alan Turing proved one result for his fellowship dissertation in 1934. See Zabell, June-July, 1995, for details.

16. Details on buffer calculations are beyond the scope of the present discussion. “As soon as possible” activity scheduling is a good rule of thumb to ensure that as much merge buffer as possible is absorbed off the critical path. Figure 6 is a modified version of an example in a white paper that shows how to handle merge buffers (also known as slack) and how to calculate a full probabilistic network without resorting to Monte Carlo Simulation. See Nevison, September, 1999, for more details.

17. In one recent study, the median project duration of 362 projects in eight companies was 8 months. If a project’s schedule is commonly in the neighborhood of 34 weeks and if it is broken into phases, it seems safe to assume that most path lengths will be covered by the rules of thumb. See Nevison, The Oak Report, February 2000.

18. Goldratt in his novel, “Critical Chain,” thinks that people pad their estimates by over estimating. Discussions over twenty years with Oak’s client project managers indicate that most technical contributors on projects under estimate. See Goldratt, 1997, and Miller, 1962, for two of the many contributors to this ongoing disagreement.


20. The two styles of negotiator are described in the work of Gerald Williams. See Williams, 1981, for details and Constructive Project Negotiation, 1998 for some illustrations.

21. Dealing with difficult negotiators is a much-studied subject. A major point is that aggressive negotiators work with a fundamentally different set of assumptions than cooperative negotiators. Perhaps the most accessible reference is Getting Past No (Ury, 1991); other references include the Constructive Project Negotiation, 1999, and Williams, 1981.

22. When 300 managers across the country were asked what factors caused problems on their projects, the most common answer was “inadequate resources.” These managers are not just whining, they are genuinely and chronically understaffed. (Taylor, 1998.) In a recent Oak Associates study, 278 project workers from ten companies said that only “seldom” was it true that “In our organization we have an adequate number of people to work on our current projects.” (Nevison, February, 2000). For more details on the challenges of multi-project management, see also Nevison, March, 2000.

23. One of the ground-breaking studies on the business value of organizational process improvements, in this case involving software projects, shows that it’s possible to get back over seven dollars for every one dollar you invest in establishing good organizational behavior. See Dion, 1993, for details.
24. Many models of business value exist in the literature. See Nevison, March, 2000 for one suggestion; see also Smith and Reinertsen, 1998; Patterson, 1993; and the “return map” of House and Price, 1991.

25. The best place to start thinking about the art and science of negotiation is Fisher, Ury, and Patton’s classic, Getting to Yes: Negotiating Agreement Without Giving In (Fisher, 1991.) Another helpful reference is Raiffa, 1982.

26. For a more extended discussion of the distinction between price and cost, see Durrenberger, 2000.

27. A new diagram to help project teams make the tradeoffs between scope, schedule, and cost (called “The 10% Chart”) is defined and described in Barker and Nevison, April, 2000.

References

About the Author

John M. (Jack) Nevison, PMP is the author of six books and numerous articles on computing and management. During the course of his business career, Nevison has built and sold two businesses, managed projects, managed project managers, and served as both an internal and external consultant to Fortune 100 companies. He is past president of the Mass Bay Chapter of the Project Management Institute (PMI®), a past president of the Greater Boston Chapter of the Association for Computing Machinery (ACM), and a Phi Beta Kappa graduate of Dartmouth.

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